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B.Sc. part-I (Sub). Date 11-04-2020
Analytical Geometry of two dimensions

Q.1: To obtain the equation of a system of co-axal circles in the simplest form.

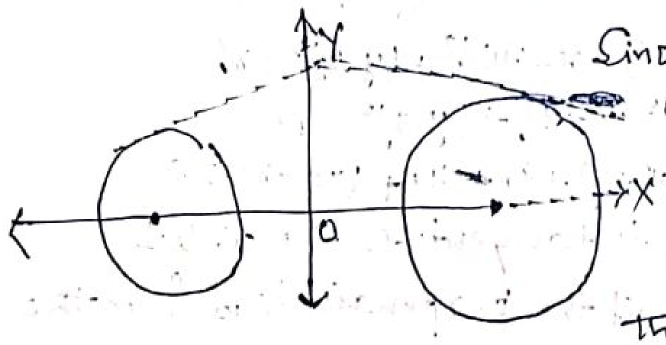
Soln: Let the two circles have for their equations

$$x^2+y^2+2g_1x+2f_1y+c_1=0 \quad \text{——— I}$$

and $$x^2+y^2+2g_2x+2f_2y+c_2=0 \quad \text{——— II}$$

Their radical axis is

$$2(g_1-g_2)x+2(f_1-f_2)y+c_1-c_2=0 \quad \text{——— III}$$



Since the common radical axis is perpendicular to the line joining the centres of any pair of circles of the system, the centres of all the circles lie on a straight line.

Let us choose the line of centres as the x-axis, then the y-coordinates of the centres of all such circles will be zero.

$$\therefore f_1=0 \text{ and } f_2=0 \quad \text{——— (IV)}$$

Again, let us take the radical axis as the axis of y, that is,

$$x=0 \quad \text{——— (V)}$$

The eqn III, from (IV) and (V), is equivalent to

$$c_1-c_2=0 \Rightarrow c_1=c_2=c \text{ (say)}$$

Hence, the circles (I) and II take the form

$$x^2+y^2+2g_1x+c=0$$
and $$x^2+y^2+2g_2x+c=0$$

These circles can be represented by

$$x^2+y^2+2gx+c=0$$

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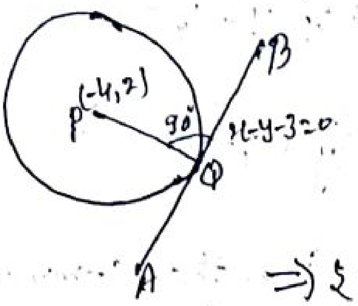
Rest part of Q.1. B.Sc-part I (Sub) Date-11-04-2020

This represents the whole system of co-axal circles.

Here, g is a variable parameter and has different values for different members of the system but c is a constant and has the same value for all the circles of the system.

Q.2. Find the equation of the family of circles having centre at $(-4,2)$. Also, find the member which has $x-y-3=0$ as a tangent.

Soln:



The equation of the family of circles having centre at $(-4,2)$ is given by

$$(x-h)^2+(y-k)^2=r^2$$

$$\Rightarrow \{x-(-4)\}^2+(y-2)^2=r^2$$

$$\Rightarrow (x+4)^2+(y-2)^2=r^2$$

$$\Rightarrow x^2+y^2+8x-4y+(20-r^2)=0$$

The line $x-y-3=0$ is tangent to the circle I

$\therefore$ The length of the perpendiculars from the centre of the circle to the given line = the radius of the circle.

$$\Rightarrow \pm\frac{-(-4)+2+3}{\sqrt{1^2+(-1)^2}}=r$$

$$\Rightarrow \pm\frac{4+2+3}{\sqrt{2}}=r \Rightarrow r=\pm\frac{9}{\sqrt{2}}$$

Substituting the value of r in I, we get

$$x^2+y^2+8x-4y+20-\frac{81}{2}=0$$

$$\Rightarrow 2(x^2+y^2)+16x-8y+40-81=0$$

$$\Rightarrow 2(x^2+y^2)+16x-8y-41=0$$

Which is the required member of the family of circles.

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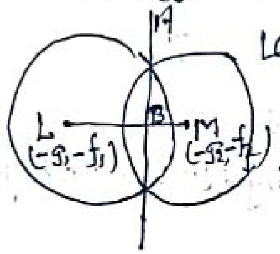
B.Sc. part-I(H), paper — II
Date — [illegible]-04-2020.

Analytical Geometry of two dimensions

Q. 23. To prove that the radical axis of two circles is perpendicular to the line joining the centres of the circles.

Proof:



Let the two circles have for their equations

$$x^2+y^2+2g_1x+2f_1y+c_1=0 \quad \text{——— (I)}$$

and

$$x^2+y^2+2g_2x+2f_2y+c_2=0 \quad \text{——— (II)}$$

The eqn of their radical axis AB is

$$2x(g_1-g_2)+2y(f_1-f_2)+c_1-c_2=0$$

The slope (or gradient) of the radical axis AB

$$= -\frac{\text{Co-efficient of } x}{\text{Coefficient of } y} = -\frac{\cancel{2}(g_1-g_2)}{\cancel{2}(f_1-f_2)} = -\frac{(g_1-g_2)}{(f_1-f_2)}$$

The co-ordinates of the centres L and M of the circles (I) and (II) are respectively $(-g_1,-f_1)$ and $(-g_2,-f_2)$

The slope of the line LM of centres

$$= \frac{\text{Difference of } y\text{-coordinates}}{\text{Difference of } x\text{-co-ordinates in the same order}}$$

$$= \frac{-f_2-(-f_1)}{-g_2-(-g_1)} = \frac{f_1-f_2}{g_1-g_2}$$

∴ The slope of AB × the slope of LM

$$= -\frac{\cancel{(g_1-g_2)}}{\cancel{f_1-f_2}} \times \frac{\cancel{f_1-f_2}}{\cancel{g_1-g_2}} = -1$$

Hence, the radical axis of two circles is perpendicular to the line joining their centres.

Proved.

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B.Sc-part-I(H). paper-II Date- 18-04-2020

Q.24. Find the equation of the circle passing through the origin and cutting the circles

$x^2+y^2-4x+6y+10=0$ and $x^2+y^2+12y+6=0$ orthogonally.

Soln: Let the ~~orthog~~ equation of the required circle be

$$x^2+y^2+2gx+2fy+c=0 \quad \text{—— (I)}$$

Since, it passes through the origin, therefore

$$0+0+2g.0+2f.0+c=0 \Rightarrow c=0$$

$\therefore$ (I) becomes: $x^2+y^2+2gx+2fy=0$ —— II

The circle II cuts the given circles $x^2+y^2+12y+6=0$ and $x^2+y^2-4x+6y+10=0$ orthogonally.

$$\therefore 2gg_1+2ff_1=c+c_1$$

$$\Rightarrow 2g.0+2f.6=0+6 \Rightarrow 12f=6 \Rightarrow f=\frac{6}{12}=\frac{1}{2}$$

and $2g.(-2)+2f.3=0+10 \Rightarrow -4g+6f=10 \Rightarrow -4g+6\times\frac{1}{2}=10$

$$\Rightarrow -4g+3=10$$

$$\Rightarrow 4g=10-3$$

$$\Rightarrow g=\frac{7}{4}$$

$$\Rightarrow g=-\frac{7}{4}$$

Substituting the values of g and f in II, we get

~~$2(x^2+y^2)-7x+2y=0$~~

$$x^2+y^2+2\times\left(-\frac{7}{4}\right)x+2\times\frac{1}{2}y+0=0 \quad [\because c=0]$$

$$x^2+y^2-\frac{7}{2}x+y=0$$

$$\Rightarrow 2(x^2+y^2)-7x+2y=0$$

This is the required equation of the circle.

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Deptt. of Mathematics
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B.Sc. part-II (H), Paper-III Date- 11-04-2020

Differential Calculus.

Q.1 If $x = \sin(\log y)$, prove that $(1-x^2)y_2 - xy_1 = y$.

Soln: We have, $x = \sin(\log y)$

$$\Rightarrow \sin^{-1}x = \log y$$

Diff. w.r. to x, we get

$$\frac{1}{\sqrt{1-x^2}} = \frac{1}{y} \cdot y_1 \Rightarrow y_1\sqrt{1-x^2} = y \qquad \left[\because y_1 = \frac{dy}{dx}\right]$$

On Squaring both sides, we get

$$y_1^2(1-x^2) = y^2$$

Diff. w.r. to x, we get

$$2y_1 y_2(1-x^2) + y_1^2(-2x) = 2yy_1$$

Dividing by $2y_1$, we get

$$y_2(1-x^2) - xy_1 = y$$

proved

Q.2. If $x = \cos(\log y)$, show that $(1-x^2)y_2 - xy_1 = y$

Soln: We have, $x = \cos(\log y) \Rightarrow \log y = \cos^{-1}x$.

~~$\Rightarrow \cos^{-1}x = \log y$~~

Diff. w.r. to x, we get

$$\frac{1}{y}\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{1}{y} \cdot y_1 = -\frac{1}{\sqrt{1-x^2}}$$

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On squaring both sides, we get

$$\frac{y_1^2}{y^2} = \frac{1}{(1-x^2)} \Rightarrow (1-x^2)y_1^2 = y^2$$

Again, differentiating w.r. to x, we get

$$(-2x)y_1^2 + (1-x^2)2y_1y_2 = 2yy_1$$

Dividing this by $2y_1$, we get

$$\Rightarrow (1-x^2)y_2 - xy_1 = y$$

Proved.

Q. 3. If $x = \cosh\left(\frac{\log y}{m}\right)$, Prove that

Soln We have, $x = \cosh\left(\frac{\log y}{m}\right)$

$$\Rightarrow \cosh^{-1}x = \frac{1}{m}\log y$$

Differentiating both sides with respect to x, we get

$$\frac{1}{\sqrt{x^2-1}} = \frac{1}{my}\cdot y_1 \qquad \left[y_1 = \frac{dy}{dx}\right]$$

$$\Rightarrow y_1\sqrt{x^2-1} = my \quad \text{—— (I)}$$

Again, differentiating both sides with respect to x, we get

$$\sqrt{x^2-1}\,y_2 + y_1\frac{1}{2\sqrt{x^2-1}}2x = my_1$$

$$\Rightarrow (x^2-1)y_2 + xy_1 = my_1\sqrt{x^2-1}$$

$$\Rightarrow (x^2-1)y_2 + xy_1 = m^2y \quad \text{, from I}$$

Hence, $(x^2-1)y_2 + xy_1 - m^2y = 0$

Q. 4. If $y = \log(\sin x)$, show that $y_3 = \frac{2\cos x}{\sin^3 x}$

Soln. Here $y = \log(\sin x)$

Diff. w.r. to x, we get

$$y_1 = \frac{1}{\sin x}\cos x = \cot x$$

Again, diff. w.r. to x, we get

$$y_2 = -\operatorname{cosec}^2 x$$

Again, diff. w.r. to x, we get

$$y_3 = -2\operatorname{cosec}x(-\operatorname{cosec}x\cot x) \Rightarrow y_3 = \frac{2\cos x}{\sin^3 x}$$ Proved

$$\left[\because 2\operatorname{cosec}^2 x\cdot\cot x = 2\frac{1}{\sin^2 x}\cdot\frac{\cos x}{\sin x} = \frac{2\cos x}{\sin^3 x}\right.$$

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Differential Calculus
Successive Differentiation
Derivatives of Higher order

Q.1. If $p = a^2\cos^2\theta + b^2\sin^2\theta$, show that

$$p + \frac{d^2p}{d\theta^2} = 2a^2 + 2b^2 - 3p.$$

Soln. We have, $p = a^2\cos^2\theta + b^2\sin^2\theta$ —— (I)

Differentiating both sides with respect to θ, we get

$$\frac{dp}{d\theta} = -2a^2\cos\theta\sin\theta + 2b^2\sin\theta\cos\theta$$
$$= -a^2\sin 2\theta + b^2\sin 2\theta$$
$$\Rightarrow \frac{dp}{d\theta} = (b^2 - a^2)\sin 2\theta \quad \text{—— II}$$

Again, differentiating both sides with respect to θ, we get

$$\frac{d^2p}{d\theta^2} = 2(b^2 - a^2)\cos 2\theta \quad \text{—— III}$$

Adding I and III, we get

$$p + \frac{d^2p}{d\theta^2} = a^2\cos^2\theta + b^2\sin^2\theta + 2(b^2 - a^2)\cos 2\theta$$
$$= a^2\cos^2\theta + b^2\sin^2\theta + 2(b^2 - a^2)(\cos^2\theta - \sin^2\theta)$$
$$= a^2\cos^2\theta + b^2\sin^2\theta + 2b^2\cos^2\theta - 2b^2\sin^2\theta - 2a^2\cos^2\theta + 2a^2\sin^2\theta$$
$$= \cos^2\theta(2b^2 - 2a^2 + a^2) + \sin^2\theta(2a^2 - 2b^2 + b^2)$$
$$= \cos^2\theta(2b^2 - 3a^2 + 2a^2) + \sin^2\theta(2a^2 - 3b^2 + 2b^2)$$
$$= (2a^2 + 2b^2)(\cos^2\theta + \sin^2\theta) - 3(a^2\cos^2\theta + b^2\sin^2\theta)$$
$$\Rightarrow p + \frac{d^2p}{d\theta^2} = 2a^2 + 2b^2 - 3p$$

Proved.

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B.Sc. part-II (Sub). Date-11-04-2020

P.2. If $ax^2+2hxy+by^2=1$, show that $y_2=\dfrac{h^2-ab}{(hx+by)^3}$

Soln We have, $ax^2+2hxy+by^2=1$ —— I

Differentiating with respect to x, we get

$$2ax+2hy+2hxy_1+2byy_1=0$$

$$\Rightarrow y_1(hx+by)=-(ax+hy)$$

$$\Rightarrow y_1=-\frac{ax+hy}{hx+by} \text{ —— II}$$

Again, differentiating II with respect to x, we get

$$y_2=-\frac{(a+hy_1)(hx+by)-(ax+hy)(h+by_1)}{(hx+by)^2}$$

$$=-\frac{ahx+aby+h^2xy_1+bhyy_1-ahx-abxy_1-h^2y-bhyy_1}{(hx+by)^2}$$

$$=-\frac{xy_1(h^2-ab)-y(h^2-ab)}{(hx+by)^2}=\frac{(h^2-ab)(y-xy_1)}{(hx+by)^2}$$

$$=\frac{h^2-ab}{(hx+by)^2}\left(y+\frac{ax^2+hxy}{hx+by}\right) \text{ [from II]}$$

$$=\frac{(h^2-ab)}{(hx+by)^2}\left(\frac{hxy+by^2+ax^2+hxy}{hx+by}\right)$$

$$\Rightarrow y_2=\frac{h^2-ab}{(hx+by)^3}(ax^2+2hxy+by^2)$$

Hence, $y_2=\dfrac{h^2-ab}{(hx+by)^3}\times 1=\dfrac{h^2-ab}{(hx+by)^3}$ [from I]

Proved.

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